The Functional Linear Mixed Model (FLMM)

The Functional Linear Mixed Model can be seen as a functional analogue of the scalar Linear Mixed Model (Laird and Ware, Biometrics, 1982) in such a way that random effects are replaced by random processes. The unit of observation is a curve.

- The General Functional Linear Mixed Model (GFLMM)

\[ Y(t) = \mu(t) + B(t) + \epsilon(t), \quad t = 1, \ldots, n, \]

- \( Y(t) \): random function observed at arguments of some set \( T \)
- \( \mu(t) \): fixed main effects surface dependent on known covariates \( x_i \)
- \( B(t) \): random functions
- \( \epsilon_i \): known covariates
- \( \epsilon(t) \): curve-specific deviations in form of smooth residual curves
- \( \epsilon_i \): white noise measurement error with variance \( \sigma_i^2(t) \)
- \( n \): number of observed curves

Assumption: \( B(t), \epsilon_i(t), \) and \( \epsilon_i \) are assumed independent for all \( i \)

Estimation

Mean, auto-covariance-, and eigenfunctions are assumed to be smooth. Dimension reduction is mandatory for estimation of functional data. We use dimension reduction via FPCA whereby the dominant modes of variation are extracted.

We face some challenges (theoretical and computational) when dealing with irregularly or even sparsely sampled data:

- PC scores cannot be estimated via numerical integration as usually done in FPCA
- smoothing of single curves may be impossible due to few measurement points
- smoothing in general is less accurate in the sparse case than for dense grid-data
- computational problems arise due to large a number of unique sampling points across curves (no Kroenecker products can be used)
- implementation is more challenging with more differenting functional data.

We extend the Functional Linear Mixed Model for longitudinal functional data of Greven, Crainiceanu, Caffo, Reich, Electronic Journal of Statistics, 2010 to more general correlated functional data which are not sampled on a fine grid and for which only a small number of measurements may be available per curve.

We first consider an FLMM with a random intercept for subjects. This is a special case of model (1) in which the correlation between measurements at the same subject is taken into account. Separate models are fit for each word group.

Results of Random Intercept Model: Results are exemplary shown for word group sch→s.

Variance decomposition: For a pre-specified level \( \alpha = 99\% \) of explained average variance, we obtain

- 4 PCs for \( B \) explaining 45.87% of the variance
- 7 PCs for \( \epsilon \) explaining 49.27% of the variance
- \( \delta^2 = 0.0027 \) which is 3.99% of the variance

Interpretation: Subjects with pos. score values for the 1st PC pronounce sound “sch” stronger than subjects with neg. score values for the 2nd PC. Differences less than the two sounds and especially have a weaker pronunciation of sound “s”.

We are currently extending model (2) by accounting for correlation between measurements of the same compound word leading to an FLMM with crossed design. Furthermore, we include the effect of a factor variable with 4 levels which indicates which syllables of the compound word \( j \) are stressed.